Closed-Form Solution to Disambiguate Defocus Blur in Single-Perspective Images

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Mathematics in Imaging

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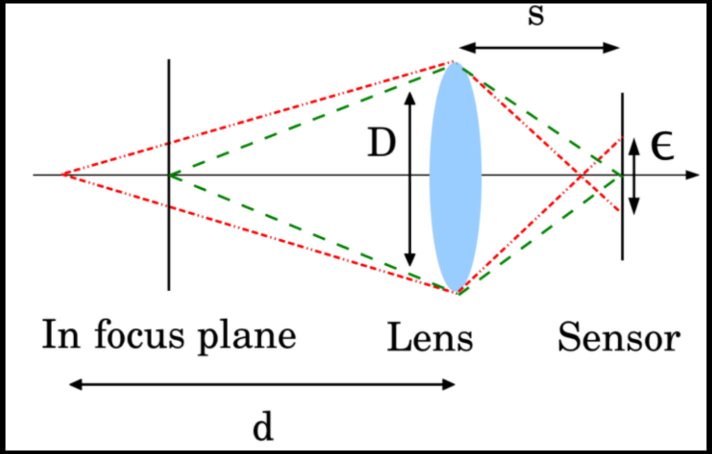
Depth and Defocus Blur

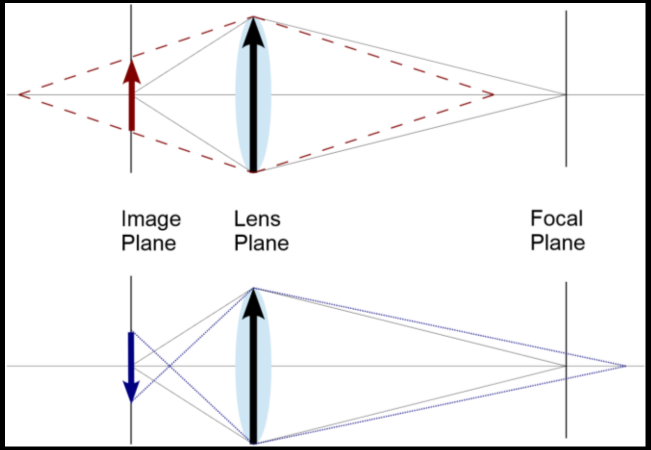
The point spread function is immediately related to the depth of the point source relative to the lens:



P. Trouvé, F. Champagnat, G. Le Besnerais, J. Sabater, T. Avignon, and J. Idier, “Passive depth estimation using chromatic aberration and a depth from defocus approach,” in Applied Optics, vol. 52, pp. 7152–7164, Optical Society of America, 2013.

Single-Perspective Depth from Defocus

Symmetric-aperture ambiguity:



Asymmetric 'apertures':

Cuttlefish

Optimized

Threshold-binarized

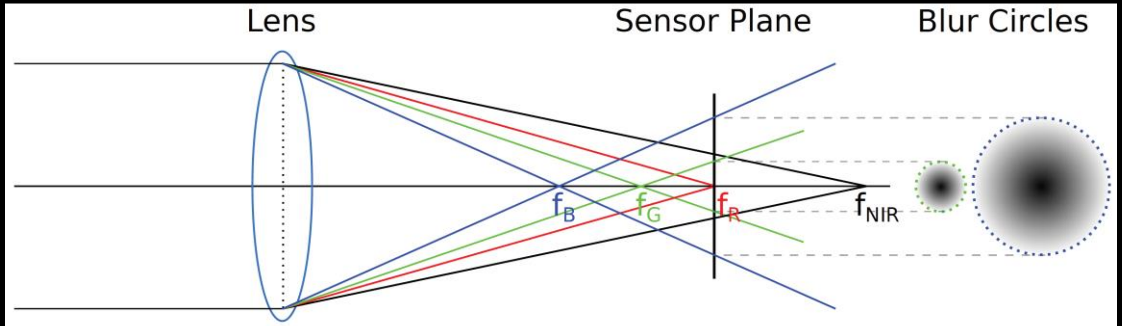
Limitations:

- Requires lens modification
- Is optimized for a set of depths
- Cannot be applied on other images
- Captured images need deblurring

A. Sellent, and P. Favaro, "Which side of the focal plane are you on?," in *IEEE ICCP*, pp. 1–8, 2014.

Electromagnetic Wavelength and Defocus Blur

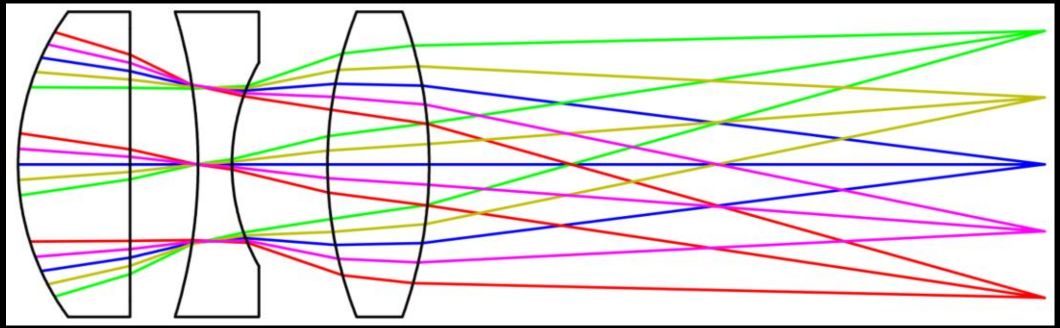
RGB-NIR PSFs on the sensor plane [1]



The power of a lens being dependent on wavelength:

$$P(\lambda) = \frac{1}{f(\lambda)} = (n(\lambda) - 1)(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2})$$

Magnified differences with a modified lens [2]



Limitations:

- The lens is modified
- Calibration is required
- Calibrated for depths (1-5m)
- Cannot be applied on other images

[1] M. El Helou, F. Dümbgen, S. Süsstrunk, "AAM: An Assessment Metric of Axial Chromatic Aberration," in *IEEE ICIP*, pp. 2486–2490, 2018.
[2] P. Trouvé, F. Champagnat, G. Le Besnerais, J. Sabater, T. Avignon, and J. Idier, "Passive depth estimation using chromatic aberration and a depth from defocus approach," in Applied Optics, vol. 52, pp. 7152–7164, Optical Society of America, 2013.

Simple Lens Modeling

Blur radius:

$$r_C(d) = L\,|1 - x/f_C + x/d|$$

Focal plane depth:

$$d_C^0 = x(x/f_C - 1)^{-1} \qquad \text{where} \qquad r_C(d_C^0) \to 0$$

Consider 2 channels A and B of different wavelength and $f_B > f_A$

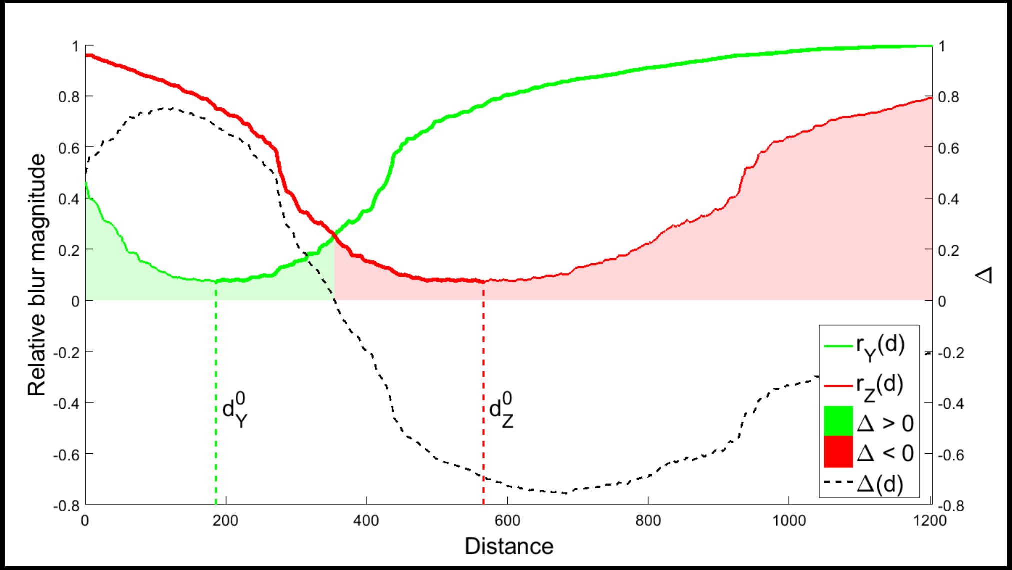
$$\Delta_{B,A}(d) \triangleq r_B(d) - r_A(d)$$

$$\Delta_{B,A}(d) = \begin{cases} \alpha \triangleq L(x/f_A - x/f_B) & d \le d_A^0 \\ 2L(1 + x/d) - L(x/f_A + x/f_B) & d \in [d_A^0, d_B^0] \\ -\alpha = L(x/f_B - x/f_A) & d \ge d_B^0 \end{cases}$$

L	simple lens aperture
x	distance lens to sensor plane
f_C	focal length for channel C
d	depth of the source point relative to the lens

Simple Lens Modeling Cont'

Illustrative simulation:



$$
\Delta_{B,A}(d) = \begin{cases} \alpha \triangleq L(x/f_A - x/f_B) & d \leq d_A^0 \\ 2L(1 + x/d) - L(x/f_A + x/f_B) & d \in [d_A^0, d_B^0] \\ -\alpha = L(x/f_B - x/f_A) & d \geq d_B^0 \end{cases}
$$

Error Analysis

Wide-band sensors:

$$\lambda_C \pm \delta_C$$

Worst case:

$$\lambda_C + \delta_C^{max}$$

Estimated radius:

$$r'_C(d) = L\left|1 - \frac{x}{f_C + \gamma_C^{max}} + \frac{x}{d}\right| + er_C$$

e_C algorithmic blur estimation error

$$E_{B,A} \triangleq er_B - er_A$$

$$\Delta'_{B,A}(d) = \begin{cases} L\left(\frac{x}{f_A + \gamma_A^{max}} - \frac{x}{f_B + \gamma_B^{max}}\right) + E_{B,A} & d \leq d_A^{0'} \\ 2L\left(1 + \frac{x}{d}\right) - L\left(\frac{x}{f_A + \gamma_A^{max}} + \frac{x}{f_B + \gamma_B^{max}}\right) + E_{B,A} & d \in [d_A^{0'}, d_B^{0'}] \\ L\left(\frac{x}{f_B + \gamma_B^{max}} - \frac{x}{f_A + \gamma_A^{max}}\right) + E_{B,A} & d \geq d_B^{0'} \end{cases}$$

Robustness Bounds

$$\Delta'_{B,A}(d) = \begin{cases} L(\frac{x}{f_A+\gamma_A^{max}} - \frac{x}{f_B+\gamma_B^{max}}) + E_{B,A} & d \leq d_A^{0'} \\ 2L(1+\frac{x}{d}) - L\left(\frac{x}{f_A+\gamma_A^{max}} + \frac{x}{f_B+\gamma_B^{max}}\right) + E_{B,A} & d \in [d_A^{0'}, d_B^{0'}] \\ L(\frac{x}{f_B+\gamma_B^{max}} - \frac{x}{f_A+\gamma_A^{max}}) + E_{B,A} & d \geq d_B^{0'} \end{cases}$$

where for channel C:

$$d_C^{0'} = x(\frac{x}{f_C+\gamma_C^{max}} - 1)^{-1}$$

New 0-crossing depth $\Delta'_{B,A}(d'_n) = 0$

$$d'_n = \frac{2Lx}{L\left(\frac{x}{f_A+\gamma_A^{max}} + \frac{x}{f_B+\gamma_B^{max}}\right) - E_{B,A} - 2L}$$

$d'_n \in [d_A^0, d_B^0]$ ➡ one-to-one mapping preserved

Evaluation with Apochromatic Lenses

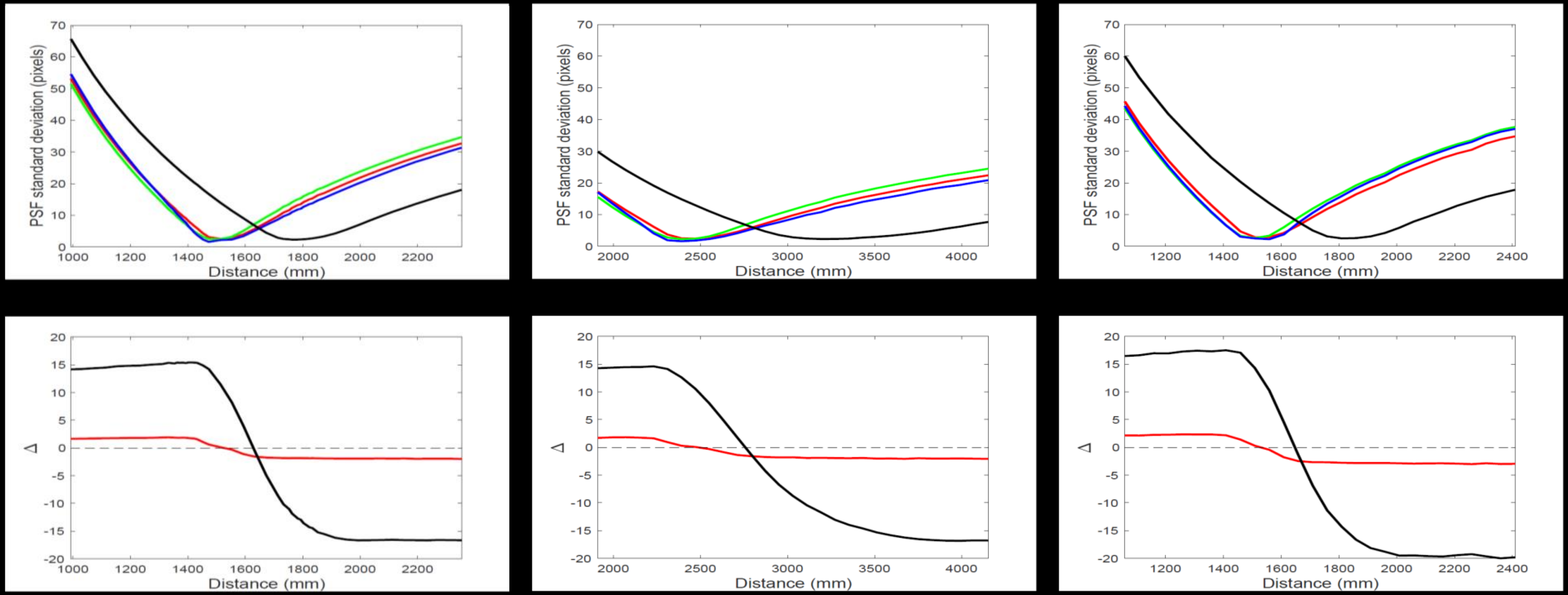
We estimate blur in controlled settings where depth is measured.

ISO 12233 Slanted Edge Standard

1. Edge Spread Function (ESF) estimated at sub-pixel precision
2. Line Spread Function (LSF) computed by ESF differentiation
3. Symmetric PSF assumption -> single LSF computed
4. PSF fitted to a 2D Gaussian distribution
5. Blur radius magnitude proportional to standard deviation

"Resolution and spatial frequency responses," Standard, Photography - Electronic still picture imaging, 2014.

Results with Apochromatic Lenses



Canon EF 50mm f/2.5 lens for the first two plots (with different focal planes) and with a Canon EF 50mm f/1.8 II lens for the third plot.

NIR in black, RGB in the respective colors.

Conclusion

Disambiguation can be done on single images, with symmetric aperture.

Disambiguation: simple lens to apochromatic lens.

NIR naturally provides more robustness (excluding superachromatic lenses).

Limitation: blur estimation is required.

Thank you

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